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Estimation in Latent Trait Models

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Estimation of ability and item parameters in latent trait models is discussed. When both ability and item parameters are considered fixed but unknown, the method of maximum likelihood for the logistic or probit models is well known. This paper discusses techniques for estimating ability and item parameters when the ability parameters, or item parameters (or both) are considered random. When the item parameters are considered fixed,

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and the ability parameters are random, from some prior distribution with fixed but unknown parameters, the EM algorithm is applied. A modification of the EM algorithm, which requires considerably less computation, is proposed. When both ability and item parameters are considered random, the EM algorithm seems to be impractical because the amount of computation needed is very large. In this case another modification to the EM algorithm is proposed. One advantage to using prior distributions is that parameter estimates usually exist in situatiosn where the maximum likelihood estimates do not. These methods are applied to the one parameter logistic or Rasch model and numerically compared using several sets of simulated data. It appears very likely that most of the methods discussed here can be readily extended to the two and three parameter logistic or probit model.

Estimation in Latent Trait Models

1. INTRODUCTION

Given that we have n subjects and k test items, consider binary responses Y_{ij} , $i = 1,\ldots, n$; $j = 1,\ldots, k$, where $Y_{ij} = 0$ or 1 depending on whether the i^{th} subjects's response to item j is incorrect or correct. Let

$$p_{ij} = 1 - q_{ij} = P(Y_{ij} = 1|\underset{\sim}{\beta}_j,\theta_i) \tag{1.1}$$

be a model for responses, where θ_i is the ability (or latent trait) parameter of the i^{th} subject and $\underset{\sim}{\beta}_j$ (possibly vector valued) is the item parameter of the j^{th} item. Given $\underset{\sim}{\theta} = (\theta_1,\ldots, \theta_n)$ and $\underset{\sim}{\beta} = (\underset{\sim}{\beta}_1,\ldots, \underset{\sim}{\beta}_k)$ we assume conditional independence among responses, $\underset{\sim}{Y} = ((Y_{ij}))$, so that

$$P(\underset{\sim}{Y} = \underset{\sim}{y}|\underset{\sim}{\theta},\underset{\sim}{\beta}) = \prod_{i=1}^{n} \prod_{j=1}^{k} p_{ij}^{y_{ij}} q_{ij}^{1-y_{ij}} \tag{1.2}$$

We wish to consider estimates of $\underset{\sim}{\theta}$ and $\underset{\sim}{\beta}$ together with measures of uncertainties in these estimates. For this purpose we introduce additional structures to the model, depending on whether we treat $\underset{\sim}{\theta}$ or $\underset{\sim}{\beta}$ (or possibly both) as fixed parameters or random with an unknown prior distribution. In the terminology commonly used in linear models analysis, we may classify the various models as shown in the following table.

		θ	
		Fixed	Random
$\underset{\sim}{\beta}$	Fixed	Fixed Effects Models	Mixed Effects Models
	Random	Mixed Effects Models	Random Effects Models

Most of the currently available techniques are for the fixed effects models, where the use of maximum likelihood for the logistic and probit models is well known (Wright and Panchapakesan 1969 and Wainer et al. 1980).

In dealing with random parameters, we shall assume that their distributions belong to certain exponential families with unknown parameters. In particular we let $\underset{\sim}{\phi}_1$ and $\underset{\sim}{\phi}_2$ denote the parameters of the prior distribution for $\underset{\sim}{\theta}$ and $\underset{\sim}{\beta}$ respectively, where $\underset{\sim}{\phi}_1$ or $\underset{\sim}{\phi}_2$ (or both) may be vector valued. When $\underset{\sim}{\theta}$ and $\underset{\sim}{\beta}$ are both random, we will further assume that they may be treated as independent random samples.

2. ESTIMATION VIA THE EM ALGORITHM

One general approach to estimating $\underset{\sim}{\theta}$ and $\underset{\sim}{\beta}$ for the random effects and mixed effects models is the EM algorithm (Dempster, Laird and Rubin 1977). The difficulty in using the EM algorithm in practice depends very much on the model. The difficulties are primarily due to the fact that the joint distribution of $(\underset{\sim}{Y},\underset{\sim}{\theta},\underset{\sim}{\beta})$ does not belong to an exponential family. We will discuss some of the difficulties and propose modifications which can be used to obtain estimates for the different models.

One way to view the EM algorithm is to consider certain parameters as nuisance parameters and integrate them out so that we are left with a likelihood function of the parameters of interest, which we can then try to maximize. The maximization is carried out interatively, by successively maximizing a function of certain unobserved sufficient statistics which are estimated by their conditional expectations given preliminary estimates of the unknown parameter.

2.1 EM Algorithm Applied to Mixed Models (MLF)

Suppose we are given k items with parameters $\underset{\sim}{\beta} = (\underset{\sim}{\beta}_1, \ldots, \underset{\sim}{\beta}_k)$ which we consider fixed, and a random sample of subjects with abilities $\underset{\sim}{\theta} = (\theta_1, \ldots, \theta_n)$, selected from a prior distribution with parameter $\underset{\sim}{\phi}_1$. In this case, $(\underset{\sim}{\beta}, \underset{\sim}{\phi}_1)$ may be considered the parameters to be estimated by the EM algorithm and $\underset{\sim}{\theta}$ an unobserved random variable with sufficient statistic $\underset{\sim}{T}_1$.

Starting with some initial estimate $(\underset{\sim}{\beta}^{(0)}, \underset{\sim}{\phi}_1^{(0)})$ for $(\underset{\sim}{\beta}, \underset{\sim}{\phi}_1)$ the algorithm repeats the following E and M steps for $\nu = 0, 1, \ldots$ until a convergence criterion is met.

E Step: Given $(\underset{\sim}{\beta}^{(\nu)}, \underset{\sim}{\phi}_1^{(\nu)})$, compute the posterior expectation of $\underset{\sim}{T}_1$,

$$t_1^{(\nu+1)} = E(\underset{\sim}{T}_1 | \underset{\sim}{Y}, \underset{\sim}{\beta}^{(\nu)}, \underset{\sim}{\phi}_1^{(\nu)})$$

M Step: Compute the value of $(\underset{\sim}{\beta}^{(\nu+1)}, \underset{\sim}{\phi}_1^{(\nu+1)})$ which maximizes

$$E(\log f(\underset{\sim}{Y}, \underset{\sim}{\theta} | \underset{\sim}{\beta}^{(\nu+1)}, \underset{\sim}{\theta}_1^{(\nu+1)}) | \underset{\sim}{Y}, \underset{\sim}{\beta}^{(\nu)}, \underset{\sim}{\theta}_1^{(\nu)})$$

where $f(\underset{\sim}{Y}, \underset{\sim}{\theta} | \underset{\sim}{\beta}^{(\nu+1)}, \phi_1^{(\nu+1)})$ is the joint probability density function of $(\underset{\sim}{Y}, \underset{\sim}{\theta})$ given $(\underset{\sim}{\beta}^{(\nu+1)}, \underset{\sim}{\phi}_1^{(\nu+1)})$.

The MLF procedure is based on the same principle as the MLF procedure for linear mixed models with normally distributed random variables discussed by Dempster, Rubin and Tsutakawa (1981).

One modification of the MLF procedure is replacing the above M Step by the following

M Step: Compute the maximum likelihood estimate of $(\underset{\sim}{\beta}, \underset{\sim}{\phi}_1)$ using $\underset{\sim}{t}_1^{(\nu+1)}$ in lieu of $\underset{\sim}{T}_1$, with $\underset{\sim}{\theta}$ fixed at its posterior expectation given $(\underset{\sim}{\beta}^{(\nu)}, \underset{\sim}{\phi}_1^{(\nu)})$.

Because this procedure conditions on the posterior expectation of $\underset{\sim}{\theta}$ given $(\underset{\sim}{\beta}^{(\nu)}, \underset{\sim}{\phi}_1^{(\nu)})$ each time through the iteration, we denote this procedure by CMLF.

We note that Sanathanan and Blumenthal (1978) use the EM algorithm to obtain estimates of the item and ability parameters for mixed effects situations. However, their procedure is somewhat different and is based on first obtaining conditional maximum likelihood (CML) estimates for $(\underset{\sim}{\beta}, \underset{\sim}{\phi}_1)$, conditional on the observed frequency distribution of raw scores, and then applying the EM algorithm to estimate $\underset{\sim}{\theta}$ while keeping $(\underset{\sim}{\beta}, \underset{\sim}{\phi}_1)$ fixed. It appears unlikely that this method generalizes to more complex models, since such conditional maximum likelihood estimates exist because of special properties of the Rasch model.

2.2 EM Algorithm Applied to Random Effects Models

Suppose we are given a random sample of item parameters $\underset{\sim}{\beta} = (\underset{\sim}{\beta}_1, \ldots, \underset{\sim}{\beta}_k)$ with prior distribution having unknown parameter $\underset{\sim}{\phi}_2$, and a random sample of subjects with ability parameter

$\underset{\sim}{\theta} = (\theta_1, \ldots, \theta_n)$ with prior distribution having unknown parameter $\underset{\sim}{\phi}_1$. Let $\underset{\sim}{T}_1$ and $\underset{\sim}{T}_2$ denote the sufficient statistics for $\underset{\sim}{\phi}_1$ and $\underset{\sim}{\phi}_2$ respectively. These statistics are unobserved, but are finite dimensional when the prior distributions belong to exponential families.

In order to apply the EM algorithm, we begin with some initial estimate of $(\underset{\sim}{\phi}_1, \underset{\sim}{\phi}_2)$, then compute in the E step,

$$(\underset{\sim}{t}_1, t_2) = E(\underset{\sim}{T}_1, \underset{\sim}{T}_2 | \underset{\sim}{Y}, \underset{\sim}{\phi}_1, \underset{\sim}{\phi}_2) \tag{2.1}$$

and, for the M step, maximize the likelihood function, for $(\underset{\sim}{\theta}, \underset{\sim}{\beta})$, with respect to $\underset{\sim}{\phi}_1$ and $\underset{\sim}{\phi}_2$, with the posterior expectation (2.1) used in place of $(\underset{\sim}{T}_1, \underset{\sim}{T}_2)$

However, for all of the latent trait models we have considered, the evaluation of (2.1) requires the numerical evaluation of multiple integrals of the order exceeding n and k. The reason for this is that the marginal posterior of θ_i and $\underset{\sim}{\beta}_j$ must be obtained through the likelihood function (1.2) which does not factor into a form suitable for low order integration.

We note however that it is considerably easier to compute the posterior expectation of $\underset{\sim}{T}_1$ when we are given $\underset{\sim}{\beta}$, and the posterior expectation of $\underset{\sim}{T}_2$ given $\underset{\sim}{\theta}$. We have thus modified the EM algorithm as follows.

Start with some initial value $(\underset{\sim}{\beta}^{(0)}, \underset{\sim}{\phi}_1^{(0)}, \underset{\sim}{\phi}_2^{(0)})$ for $(\underset{\sim}{\beta}, \underset{\sim}{\phi}_1, \underset{\sim}{\phi}_2)$, and repeat the following for $\nu = 0, 1, \ldots,$ until a convergence criterion is satisfied.

E_1 Step: Compute

$$\underset{\sim}{\theta}^{(\nu+1)} = E(\underset{\sim}{\theta} | \underset{\sim}{Y}, \underset{\sim}{\phi}_1^{(\nu)}, \underset{\sim}{\beta}^{(\nu)}) \tag{2.2}$$

$$\underset{\sim}{t}_1^{(\nu+1)} = E(\underset{\sim}{T}_1 | \underset{\sim}{Y}, \underset{\sim}{\phi}_1^{(\nu)}, \underset{\sim}{\beta}^{(\nu)}) \tag{2.3}$$

E_2 Step: Compute

$$\underset{\sim}{\beta}^{(\nu+1)} = E(\underset{\sim}{\beta} | \underset{\sim}{Y}, \underset{\sim}{\phi}_2^{(\nu)}, \underset{\sim}{\theta}^{(\nu+1)}) \tag{2.4}$$

$$\underset{\sim}{t}_2^{(\nu+1)} = E(\underset{\sim}{T}_2 | \underset{\sim}{Y}, \underset{\sim}{\phi}_2^{(\nu)}, \underset{\sim}{\theta}^{(\nu+1)}) \tag{2.5}$$

M_1 Step: Compute $\underset{\sim}{\phi}_1^{(\nu+1)}$, the maximum likelihood estimator of $\underset{\sim}{\phi}_1$ using $\underset{\sim}{t}_1^{(\nu+1)}$ in place of $\underset{\sim}{T}_1$.

M_2 Step: Compute $\underset{\sim}{\phi}_2^{(\nu+1)}$, the maximum likelihood estimator of $\underset{\sim}{\phi}_2$ using $\underset{\sim}{t}_2^{(\nu+1)}$ in place of $\underset{\sim}{T}_2$.

If convergence is attained the terminal value of $(\theta^{(\nu)}, \beta^{(\nu)}, \phi_1^{(\nu)}, \phi_2^{(\nu)})$ will satisfy the consistency conditions

$$E(\underset{\sim}{T}_1 | \underset{\sim}{Y}, \underset{\sim}{\phi}_1, \underset{\sim}{\beta}) = E(\underset{\sim}{T}_1 | \underset{\sim}{\phi}_1, \underset{\sim}{\beta}) \tag{2.6}$$

and

$$E(\underset{\sim}{T}_2 | \underset{\sim}{Y}, \underset{\sim}{\phi}_2, \underset{\sim}{\theta}) = E(\underset{\sim}{T}_2 | \underset{\sim}{\phi}_2, \underset{\sim}{\theta}) . \tag{2.7}$$

Note that equation (2.3) is similar to the E Step of the MLF procedure for the mixed model, with the exception that we condition on the posterior expectation of $\underset{\sim}{\beta}$ rather than on the maximum likelihood estimate.

The estimates $(\underset{\sim}{\phi}_1^{(\nu)}, \underset{\sim}{\phi}_2^{(\nu)})$ thus obtained are not true maximum likelihood estimates, which would result if straight EM were possible. Because of the conditional nature of this solution, and because both $\underset{\sim}{\theta}$ and $\underset{\sim}{\beta}$ are random, we denote this procedure by CMLR.

The assumption that $\underset{\sim}{\beta}$ is a random sample from some common distribution could be unrealistic when item pools are deliberately organized to contain a wide spectrum of difficulties or when other differences are present. One Bayesian solution to this problem is to consider a uniform prior distribution on each $\underset{\sim}{\beta}_i$ where the range is, in principle, finite but very large. Using an algorithm similar to CMLR, the posterior distribution of $\underset{\sim}{\beta}$ (conditional on $\underset{\sim}{\theta}$) can be computed and used to compare different items. This procedure will be denoted by CMLU and is illustrated below.

3. APPLICATION OF EM ALGORITHM TO RASCH MODEL

Given θ_i and β_j , the Rasch model, or one parameter logistic model, gives the probability distribution of Y_{ij} as

$$P(Y_{ij}=y_{ij} \mid \theta_i, \beta_j) = \frac{\exp(y_{ij}(\theta_i - \beta_j))}{1 + \exp(\theta_i - \beta_j)} , \quad y_{ij} = 0, 1 .$$

In the Rasch model, θ_i is called the ability parameter and β_j is called the item or difficulty parameter. Assuming conditional independence among the responses $\underset{\sim}{Y} = ((Y_{ij}))$, the probability distribution of $\underset{\sim}{Y}$ can be written as

$$P(\underset{\sim}{Y}=\underset{\sim}{y}|\underset{\sim}{\theta},\underset{\sim}{\beta}) = \prod_{i=1}^{n}\prod_{j=1}^{k}\frac{\exp(y_{ij}(\theta_i-\beta_j))}{1+\exp(\theta_i-\beta_j)} \tag{3.1}$$

$$= \frac{\exp\left(\sum_{i=1}^{n} r_i\theta_i - \sum_{j=1}^{k} q_j\theta_j\right)}{\prod_{i=1}^{n}\prod_{j=1}^{k}(1+\exp(\theta_i-\beta_j))}$$

where r_i is the raw score of the i^{th} examinee defined by

$$r_i = \sum_{j=1}^{k} y_{ij}$$

and q_j is the item score for the j^{th} item defined by

$$q_j = \sum_{i=1}^{n} y_{ij} .$$

3.1. MLF Estimation

For MLF, we assume that $\theta_1, \ldots, \theta_n$ form a random sample of size n from the normal distribution with mean μ and variance σ^2, where μ and σ^2 are fixed but unknown quantities. The difficulty parameters $\underset{\sim}{\beta} = (\beta_1,\ldots,\beta_k)$ are also assumed to be fixed but unknown. Since $\theta_1, \ldots, \theta_n$ are assumed independent, the prior distirbution $p(\theta|\mu,\sigma)$ of $\underset{\sim}{\theta} = (\theta_1,\ldots,\theta_n)$ is the product of n normal distributions, each with mean μ and variance σ^2. From (3.1), the likelihood function of $\underset{\sim}{\theta}$, given $\underset{\sim}{\beta}$, is

$$\ell(\underset{\sim}{\theta}|\underset{\sim}{y},\underset{\sim}{\beta}) = P(\underset{\sim}{Y}=\underset{\sim}{y}|\underset{\sim}{\theta},\underset{\sim}{\beta}) .$$

Combining the prior distribution of $\underset{\sim}{\theta}$, $p(\underset{\sim}{\theta}|\mu,\sigma)$, with the likelihood function of $\underset{\sim}{\theta}$, $\ell(\underset{\sim}{\theta}|\underset{\sim}{y},\underset{\sim}{\beta})$, we can obtain the posterior distribution of $\underset{\sim}{\theta}$, given $\underset{\sim}{\beta}$, which is

$$p(\underset{\sim}{\theta}|\underset{\sim}{y},\mu,\sigma,\underset{\sim}{\beta}) = Hp(\underset{\sim}{\theta}|\mu,\sigma)\ell(\underset{\sim}{\theta}|\underset{\sim}{y},\underset{\sim}{\beta}) \tag{3.2}$$

where H is the normalizing constant chosen such that the expression on the right side of (3.2) integrates to one. By integrating (3.2) with respect to $\theta_1, \ldots, \theta_{i-1}, \theta_{i+1}, \ldots, \theta_n$, we can obtain the marginal posterior distribution of θ_i , which can be written as

$$p(\theta_i|\underset{\sim}{y},\mu,\sigma,\underset{\sim}{\beta}) = \frac{H_i\exp((-(\theta_i-\mu)^2/2\sigma^2)+r_i\theta_i)}{\prod_{j=1}^{k}(1+e^{\theta_i-\beta_j})}$$

where H_i is the appropriate nomralizing constant.

The estimation of ability and difficulty parameters proceeds as follows. Begin with an initial set of estimates, $\underset{\sim}{\beta}^{(0)} = (\beta_1^{(0)}, \ldots, \beta_k^{(0)})$, for the item parameters, and initial estimates $\mu^{(0)}$ and $\sigma^{(0)}$ for μ and σ respectively. A convenient choice for initial estimates of the difficulty parameters is the negative of the standardized item scores. Then for $\nu = 0, 1, \ldots$, until a convergence criterion is satisfied, repeat the E and M steps.

E Step: Calculate

$$t_{11} = \sum_{i=1}^{n} \theta_{i1}^{(\nu+1)} \tag{3.3}$$

$$t_{12} = \sum_{i=1}^{n} \theta_{i2}^{(\nu+1)} \tag{3.4}$$

where

$$\theta_{i1}^{(\nu+1)} = E(\theta_i | \underset{\sim}{Y}, \mu^{(\nu)}, \sigma^{(\nu)}, \underset{\sim}{\beta}^{(\nu)}) \tag{3.5}$$

and

$$\theta_{i2}^{(\nu+1)} = E(\theta_i^2 | \underset{\sim}{Y}, \mu^{(\nu)}, \sigma^{(\nu)}, \underset{\sim}{\beta}^{(\nu)}) \tag{3.6}$$

M Step: Find the values of $\mu^{(\nu+1)}$, $\sigma^{(\nu+1)}$ and $\underset{\sim}{\beta}^{(\nu+1)}$ which maximize

$$E(\log p(\underset{\sim}{\theta} | \underset{\sim}{Y}, \mu^{(\nu+1)}, \sigma^{(\nu+1)}, \beta^{(\nu+1)}) | \mu^{(\nu)}, \sigma^{(\nu)}, \underset{\sim}{\beta}^{(\nu)}) \tag{3.7}$$

In order to assure uniqueness of the parameterization, after each M-step, we standardize the difficulty parameters so that they sum to zero.

Since $\exp(-(\theta_i-\mu)^2/2\sigma^2)$ is in the integrand in (3.5) and (3.6), a simple change of variable will put this into a form where Gauss-Hermite quadrature formulas for numerical integration are suitable. To obtain the values of $\mu^{(\nu+1)}$ and $\sigma^{(\nu+1)}$ which maximize (3.7), we differentiate (3.7) with respect to $\mu^{(\nu+1)}$ and $\sigma^{(\nu+1)}$ and set these results equal to zero. The integral in (3.7) can be written as the sum of a finite number of single integrals, each of which is uniformly convergent in $\mu^{(\nu+1)}$ and $\sigma^{(\nu+1)}$, hence moving the differentiation operator inside the integral is valid. This yields simple and familiar expressions for the $\mu^{(\nu+1)}$ and $\sigma^{(\nu+1)}$ which maximize (3.7), namely,

$$\mu^{(\nu+1)} = t_{11}/n \tag{3.8}$$

and

$$\sigma^{(\nu+1)^2} = t_{12}/n - \mu^{(\nu+1)^2} \tag{3.9}$$

To find the $\underset{\sim}{\beta}$ that maximizes (3.7), we differentiate (3.7) with respect to $\beta_j^{(\nu+1)}$, $j = 1,\ldots,k$ and set these results equal to zero. Again, it is valid to differentiate inside the integral, but now we cannot get a closed form expression for $\beta_j^{(\nu+1)}$. Instead, we get k nonlinear equations

$$-q_j + \sum_{i=1}^{n} \int_{-\infty}^{\infty} \frac{\exp(\theta_i - \beta_j^{(\nu+1)})}{1 + \exp(\theta_i - \beta_j^{(\nu+1)})} p(\theta_i | \underset{\sim}{y}, \mu, \sigma, \underset{\sim}{\beta}^{(\nu)}) d\theta_i = 0 , \tag{3.10}$$

$j = 1,\ldots, k$. These equations can be solved one at a time by the secant method described in Conte and deBoor (1972).

3.2 CMLF Applied to Rasch Model

The MLF procedure can be modified slightly by doing the following. As before, begin with initial estimates $\mu^{(0)}, \sigma^{(0)}$ and $\underset{\sim}{\beta}^{(0)}$ for μ, σ and $\underset{\sim}{\beta}$. Then, until a convergence criterion is satisfied, for $\nu = 0, 1, \ldots,$ repeat the following steps:

E Step: Calculate $\underset{\sim}{t}_1 = (t_{11}, t_{12})$ as in (3.3) and (3.4)

M_1 Step: Using $\underset{\sim}{\theta}^{(\nu+1)}$ as the actual values of $\underset{\sim}{\theta}$, calculate the maximum likelihood estimate of $\underset{\sim}{\beta}$.

M_2 Step: Set $\mu^{(\nu+1)}$ and $\sigma^{(\nu+1)^2}$ equal to the values given in (3.8) and (3.9) respectively.

After each M_2 step, we standardize the item parameters so that they sum to zero. To do the M step, we find the log-likelihood function of $\underset{\sim}{\beta}$ given $\underset{\sim}{y}$ and $\underset{\sim}{\theta}^{(\nu+1)}$ to be

$$L(\underset{\sim}{\beta}|\underset{\sim}{y},\underset{\sim}{\theta}^{(\nu+1)}) = \sum_{i=1}^{n} \theta_i^{(\nu+1)} r_i - \sum_{j=1}^{k} \beta_j q_j - \sum_{i=1}^{n} \sum_{j=1}^{k} \log(1+\exp(\theta_i^{(\nu+1)} - \beta_j)) \quad . \tag{3.11}$$

Differentiating (3.11) with respect to β_j , and setting the result equal to zero yields a nonlinear equation whose root is the maximum likelihood estimate of β_j , when $\underset{\sim}{\theta}$ is given. That is, we numerically solve the equation

$$\frac{\partial L}{\partial \beta_j} = -q_j + \sum_{i=1}^{n} (1+\exp(\beta_j - \theta_i^{(\nu+1)})) = 0$$

for β_j. If q_j is not zero or k , then this equation will have a unique solution.

3.3 CMLR Applied to Rasch Model

Suppose now that $\theta_1, \ldots, \theta_n$ is a random sample from the normal distribution with mean μ and variance σ^2 , and $\beta_1, \ldots,$ β_k is a random sample from the normal distribution with mean zero and variance τ^2. Again, we start with initial estimates $\underset{\sim}{\beta}^{(0)}$, $\mu^{(0)}$, $\sigma^{(0)}$ and $\tau^{(0)}$ for $\underset{\sim}{\beta}, \mu, \sigma$ and τ respectively. For $\nu = 0, 1, \ldots$, until a convergence criterion is reached, we repeat the following steps:

E_1 Step: Calculate $\underset{\sim}{t}_1 = (t_{11}, t_{12})$ as in (3.3) and (3.4).

E_2 Step: Calculate $\underset{\sim}{t}_2 = (t_{21}, t_{22})$ by

$$t_{21} = \sum_{j=1}^{k} \beta_{j1}^{(\nu+1)}$$

$$t_{22} = \sum_{j=1}^{k} \beta_{j2}^{(\nu+1)}$$

where

$$\beta_{j1}^{(\nu+1)} = E(\beta_j | Y, \tau^{(\nu)^2}, \underset{\sim}{\theta}^{(\nu+1)}) \tag{3.12}$$

and

$$\beta_{j2}^{(\nu+1)} = E(\beta_j^2 | Y, \tau^{(\nu)^2}, \underset{\sim}{\theta}^{(\nu+1)}) \ . \tag{3.13}$$

M Step: Set $\mu^{(\nu+1)}$ and $\sigma^{(\nu+1)^2}$ equal to the values given in (3.8) and (3.9) respectively, and set

$$\tau^{(\nu+1)} = t_{22}/k - (t_{21}-k)^2$$

After each M step, we standardize the item scores so that they sum to zero. Since $\beta_1, \ldots, \beta_k$ are independent and normally distributed, the joint distribution is the product of k normal distributions each with mean zero and variance τ^2.

Combining the likelihood function of $\underset{\sim}{\beta}$ with the prior distribution of $\underset{\sim}{\beta}$ we obtain the posterior distribution of $\underset{\sim}{\beta}$. Integrating with respect to $\beta_1, \ldots, \beta_{j-1}, \beta_{j+1}, \ldots, \beta_k$, yields the marginal posterior distribution of β_j given $\underset{\sim}{\theta}$,

$$p(\beta_j | \underset{\sim}{y}, \tau, \underset{\sim}{\theta}) = \frac{G_j \exp(-\beta_j^2/2\tau^2 - \beta_j q_j)}{\prod_{i=1}^{n} (1+\exp(\theta_i - \beta_j))} \tag{3.14}$$

where G_j is the appropriate normalizing constant. For evaluating posterior moments, here again Gauss-Hermite quadrature formulas are applicable.

CMLU is a limiting case of CMLR where the prior distribution of the item parameters is taken to be uniform. When the β's are independent and have a uniform prior, the posterior distribution

of β_j can be written as

$$p(\beta_j|\underset{\sim}{y},\tau,\underset{\sim}{\theta}) = \frac{F_j \exp(-\beta_j q_j)}{\prod_{i=1}^{n} (1+\exp(\theta_i-\beta_j))} \tag{3.15}$$

where F_j is the appropriate normalizing constant. If q_j is not zero or k, then F_j can be chosen to make this integrate to one, and also, moments of all order exist for β_j . The estimation procedure is similar to that of CMLR except that, first, the posterior distribution of β_j is taken to be the expression given in (3.15), and second, the estimate for $\tau^{(\nu+1)}$ need not be computed.

4. NUMERICAL EXAMPLES

In this section we discuss the implementation of these procedures to four simulated data sets. In all four sets, the item parameters were taken to be standard normal random variates. In two of the data sets, denoted SI and SII, the ability parameters were taken as standard normal random variates. In the third data set, denoted SIII, the ability parameters were taken as a random sample from the uniform distribution on the interval from -3 to 3. The ability parameters for the fourth simulated data set, were taken as random variates from the Cauchy distribution, which has probability density function

$$f(x) = \frac{10}{\pi(1+100x^2)} , -\infty < x < \infty.$$

In all four cases, the size of the data sets were 100 examinees and 45 items.

We estimated the ability and difficulty parameters by the five methods: maximum likelihood (ML), MLF, CMLF, CMLR, and CMLU. In the data set SI, one raw score was k (45) and in data set SIV, one raw score was zero. In these cases the maximum likelihood estimate for the ability of the subject scoring perfectly or scoring a zero, does not exist. Thus, we did not apply ML in these two cases.

The estimated parameters μ, σ and τ for each of the four data sets are shown in Table 1. In some models, the three parameters μ, σ and τ do not all appear. When this happens, we have given the values of the appropriate sample statistic and put these numbers in parentheses. The sample statistics of the actual ability and item parameters are also given.

In most cases the estimates for μ and σ obtained by the MLF and CMLF methods were quite close to each other and quite close to the estimates obtained by the CMLU methods. The ML estimates were somewhat close to the MLF, CMLF and CMLU estimates. The CMLR estimates were usually quite far from the estimates obtained by the other methods. In one extreme case, τ , in the CMLR method actually converged to zero, meaning that the estimates of all item parameters were zero. Still estimates for μ, σ and $\underset{\sim}{\theta}$ were obtained in this case.

Figures 1 through 4 give scatter plots of the ML estimates of $\underset{\sim}{\theta}$ for SII on the vertical axes, and MLF, CMLF, CMLR and CMLU estimates on the horizontal axes. Figures 5 through 8 give scatter plots for the corresponding item parameters.

The plots in Figures 1 through 4 show the relation between the sets of ability estimates. The estimates obtained by ML were more spread out than the estimates from the other four methods. Expecially noticeable is the way in which the MLF, CMLF, CMLR and CMLU pulled the estimates at the extreme ends closer to zero.

The plots in Figure 5, 6, and 8 show a nearly linear relationship between the ML estimates and the MLF, CMLF and CMLU estimates of the item parameters that lies on the diagonal line through the origin. The plot of ML versus CMLR in Figure 7 shows a nearly linear relationship, except here the CMLR estimates are much more spread out than the ML estimates. The estimate for τ in SII was 2.6401 which accounts for the large variation in the CMLR estimates.

Since the data was simulated, the actual values of $\underset{\sim}{\theta}$ and $\underset{\sim}{\beta}$ were known, so these can be compared with the estimates. Table 2 shows the mean squared errors (MSE's) for the different estimation techniques. An asterisk next to a value indicates that the MSE for that method was smallest among the five methods. In most cases the MSE's from the MLF, CMLF and CMLU were very close. In five of the eight cases, the MSE from the CMLF was the lowest among the five methods. The MSE's for CMLR in Table 2 are generally larger than for other methods. This may be due to the poorer estimates of μ, σ and τ as seen in Table 1.

5. SUMMARY AND FURTHER REMARKS

We have discussed several methods for estimating parameters in the Rasch model, namely, MLF, CMLF, CMLR, and CMLU. In all four of these methods, the ability parameter of a subject can be

estimated even when that subject scores perfectly or scores a zero, a property not shared by maximum likelihood. If an item score for some item is either zero or n, then the difficulty parameter for this item cannot be estimated by the MLF, CMLF, or CMLU procedures. However this parameter can be estimated if the CMLR procedure is used.

Since the item parameters are estimated one at a time (in all four methods discussed here), it is feasible that these methods could be extended to a two or three parameter logistic model. In extending the CMLR or CMLU procedure, it is necessary to calculate double integrals for the two parameter model and triple integrals for the three parameter model, for each item in the test, each time through the iteration. It might be practical to compute double integrals, however the computer time necessary to do triple integrals would probably be prohibitive. On the other hand, when extending the MLF or CMLF procedures, it is necessary to maximize functions of two or three variables. The Newton-Raphson technique is a practical way to do this even for a three parameter logistic model.

Table 1. Estimates of Parameters of Prior Distribution

		μ	σ	τ
	ACTUAL	(-0.1177)	(1.0388)	(1.0245)
	ML	NA	NA	NA
SI	MLF	-0.1452	1.0460	(1.0040)
	CMLF	-0.1447	1.0407	(0.9879)
	CMLR	-0.1299	0.9092	0.4926
	CMLU	-0.1451	1.0442	(0.9991)
	ACTUAL	(-0.0357)	(0.9758)	(1.0496)
	ML	(-0.0953)	(1.0735)	(1.1430)
SII	MLF	-0.0916	0.9764	(1.1142)
	CMLF	-0.2894	0.5046	(1.0949)
	CMLR	-0.4762	0.9595	2.6401
	CMLU	-0.2904	0.5070	(1.1080)
	ACTUAL	(-0.1878)	(1.8103)	(0.9381)
	ML	(-0.1704)	(1.8765)	(0.9352)
SIII	MLF	-0.1711	1.8040	0.9071
	CMLF	-0.1710	1.8035	(0.9059)
	CMLR	-0.1517	1.5743	0.
	CMLU	-0.1712	1.8055	(0.9103)
	ACTUAL	(-0.3759)	(1.1151)	(0.8796)
	ML	NA	NA	NA
SIV	MLF	-0.2904	0.5075	(0.9252)
	CMLF	-0.2894	0.5046	(0.9110)
	CMLR	-0.4762	0.9595	2.6401
	CMLU	-0.2904	0.5070	(0.9232)

NA - method not applicable in this case.

TABLE 2. MSE's of Ability and Item Parameters

		$\frac{1}{100}\sum_{i=1}^{100}(\theta_i-\hat{\theta}_i)^2$	$\frac{1}{45}\sum_{j=1}^{45}(\beta_j-\hat{\beta}_j)^2$
	ML	NA	NA
	MLF	.11486	.04059*
SI	CMLF	.11473*	.04083
	CMLR	.12955	.30639
	CMLU	.11481	.04069
	ML	.13247	.06626
	MLF	.10541	.06049
SII	CMLF	.10540*	.05749*
	CMLR	.12145	.21690
	CMLU	.10542	.05982
	ML	.20119	.07112
	MLF	.16138	.07005
SIII	CMLF	.16123	.06992*
	CMLR	.21943	.86055
	CMLU	.16119*	.07002
	ML	NA	NA
	MLF	.31587*	.06347
	CMLF	.72997	.06142*
	CMLR	.54443	2.94153
	CMLU	.72788	.06303

NA - method not applicable in this case.
* - method had lowest MSE among five methods.

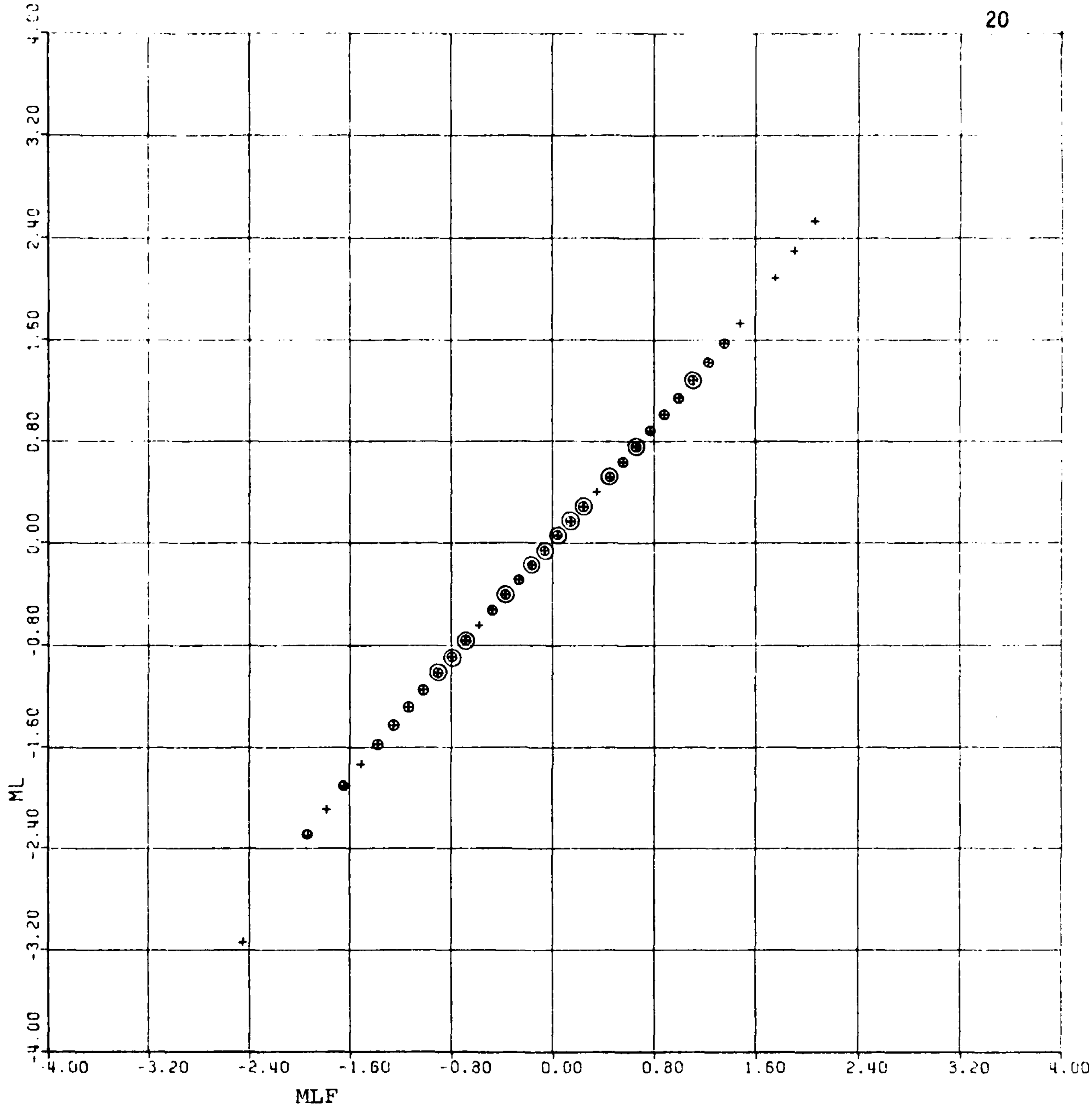

FIGURE 1. ML vs MLF Estimates of Ability

+ - one observation
⊕ - two or three observations
◎ - four or more observations

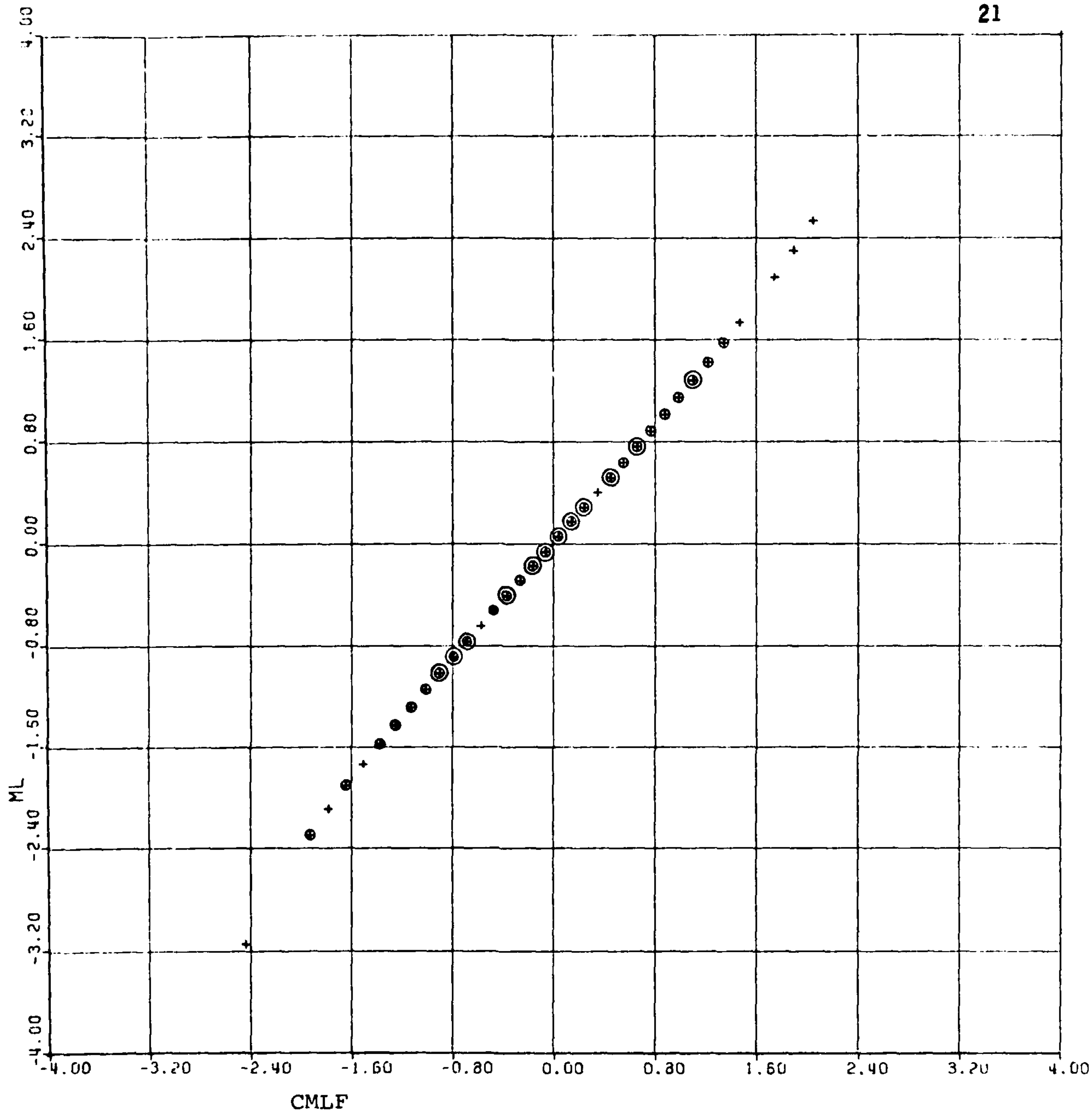

FIGURE 2. ML vs CMLF Estimates of Ability

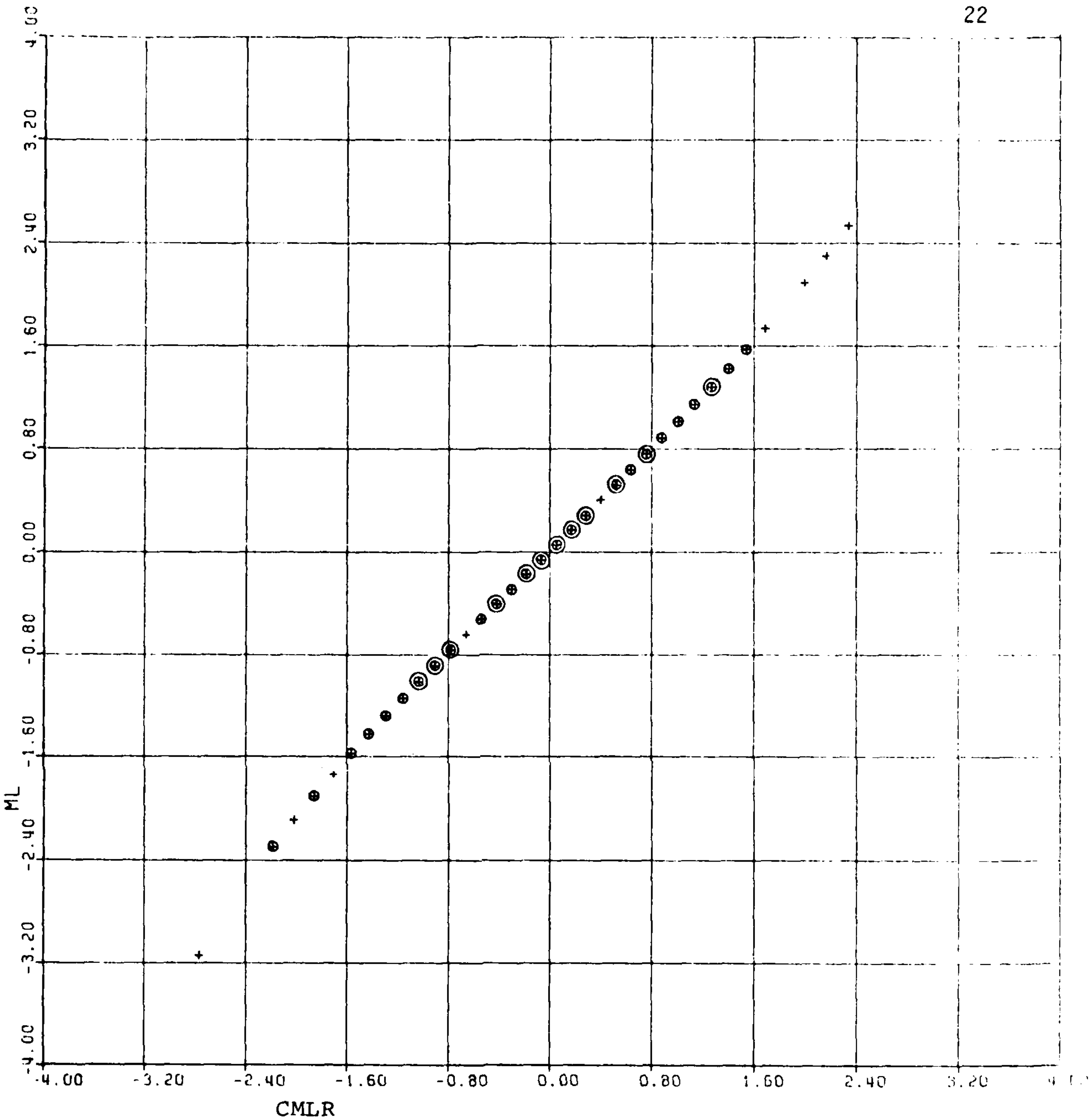

FIGURE 3. ML vs CMLR Estimates of Ability

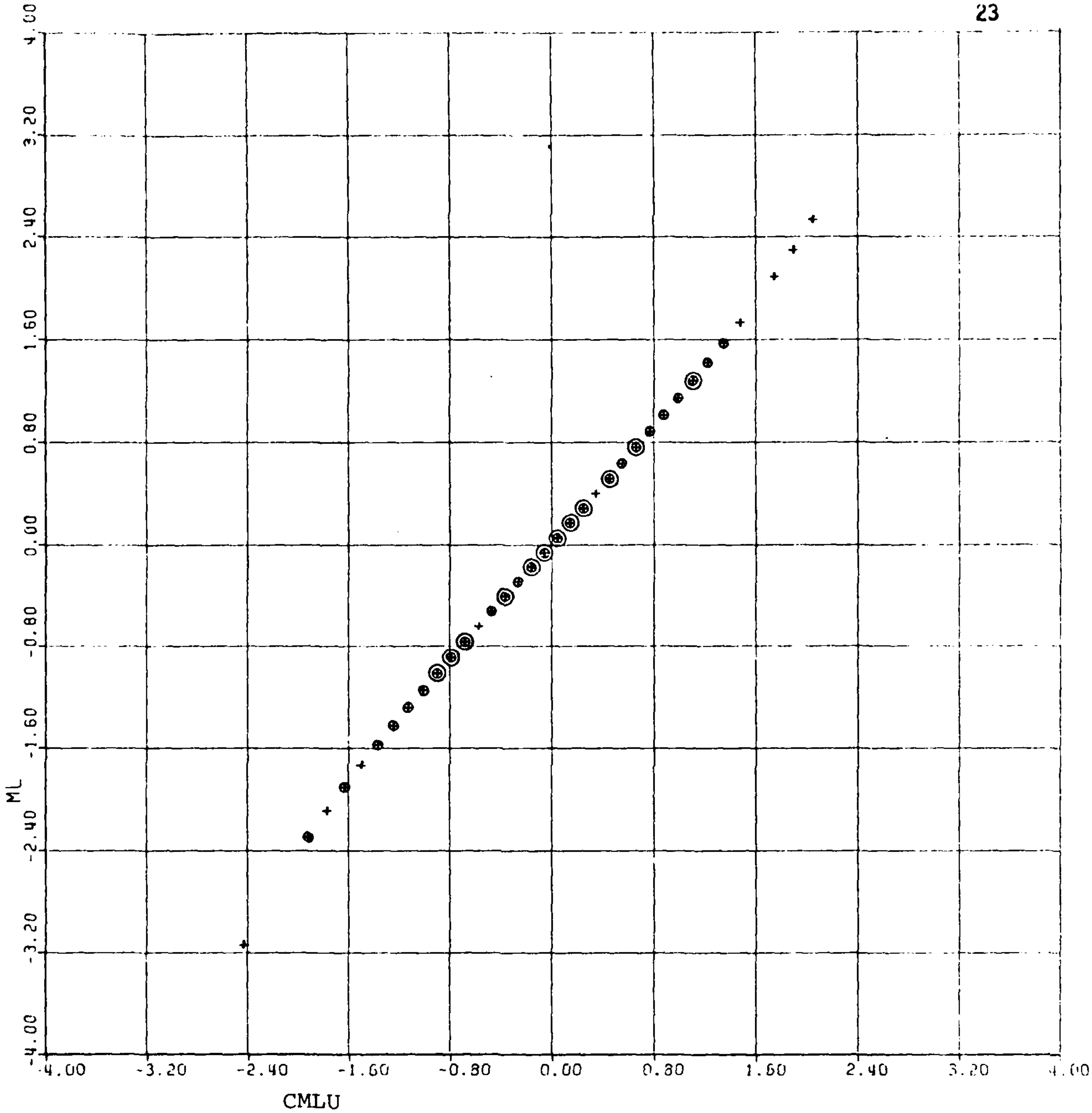

FIGURE 4. ML vs CMLU Estimates of Ability

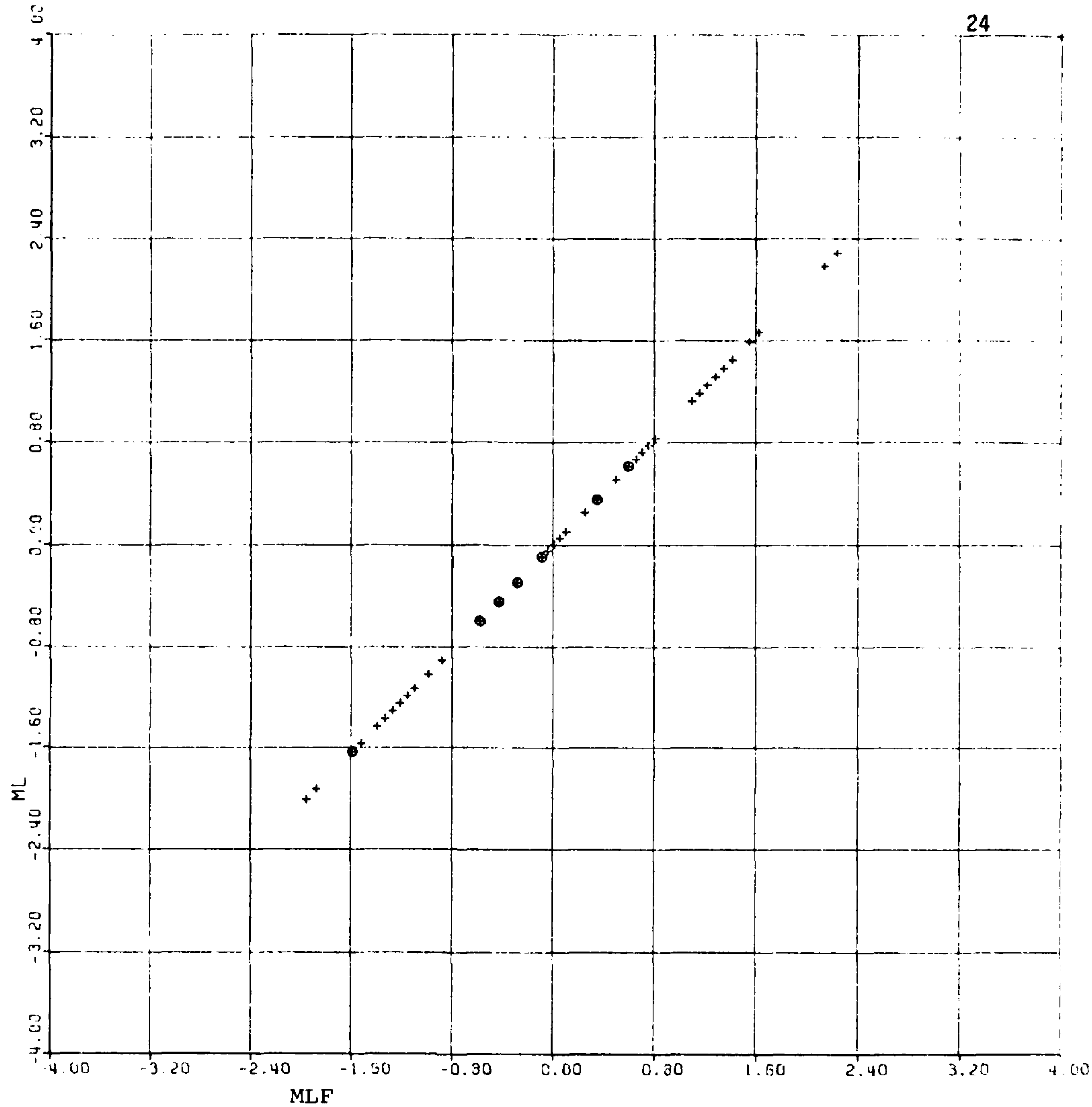

FIGURE 5. ML vs MLF Estimates of Difficulty Parameters

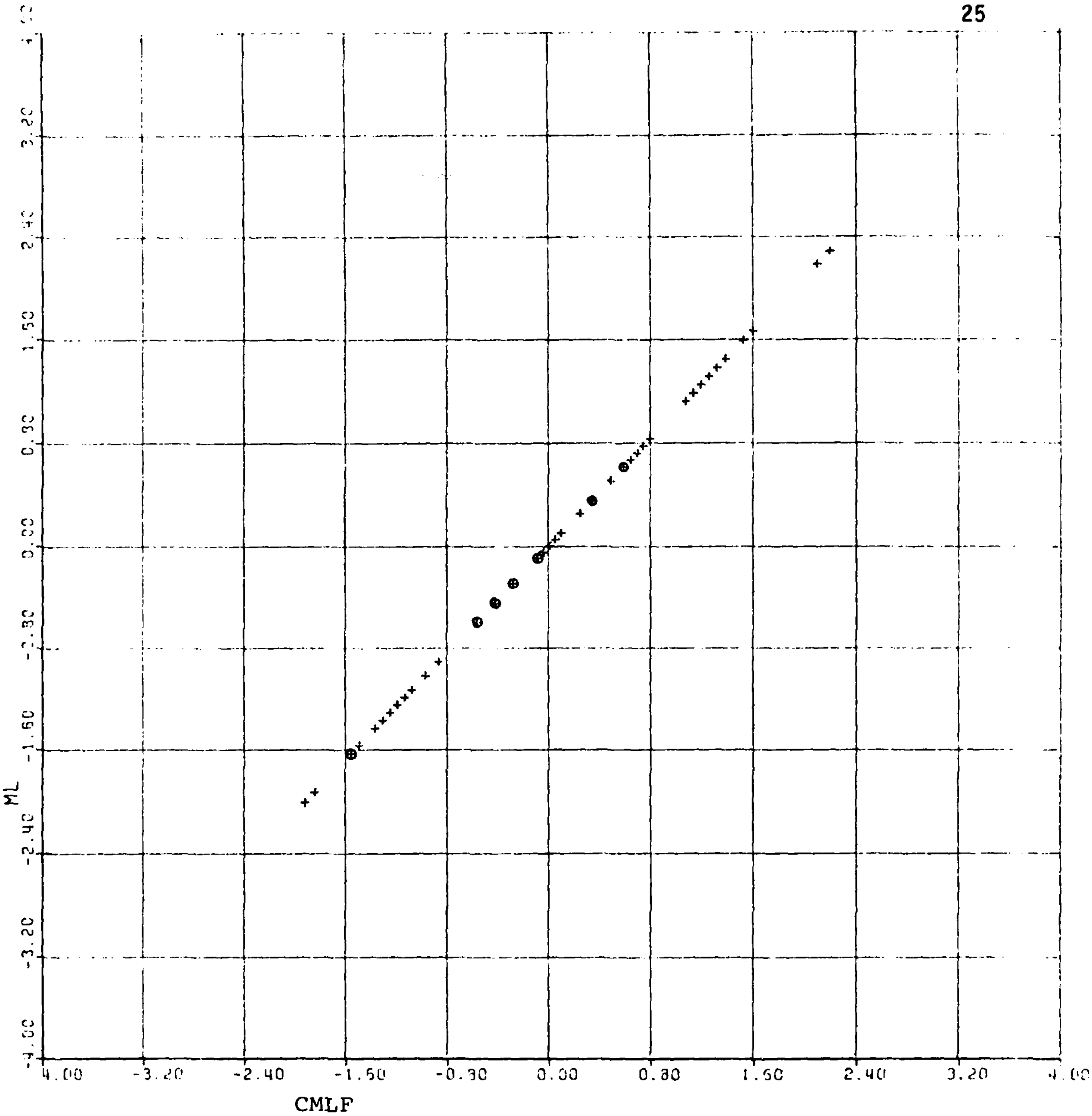

FIGURE 6. ML vs CMLF Estimates of Difficulty Parameters

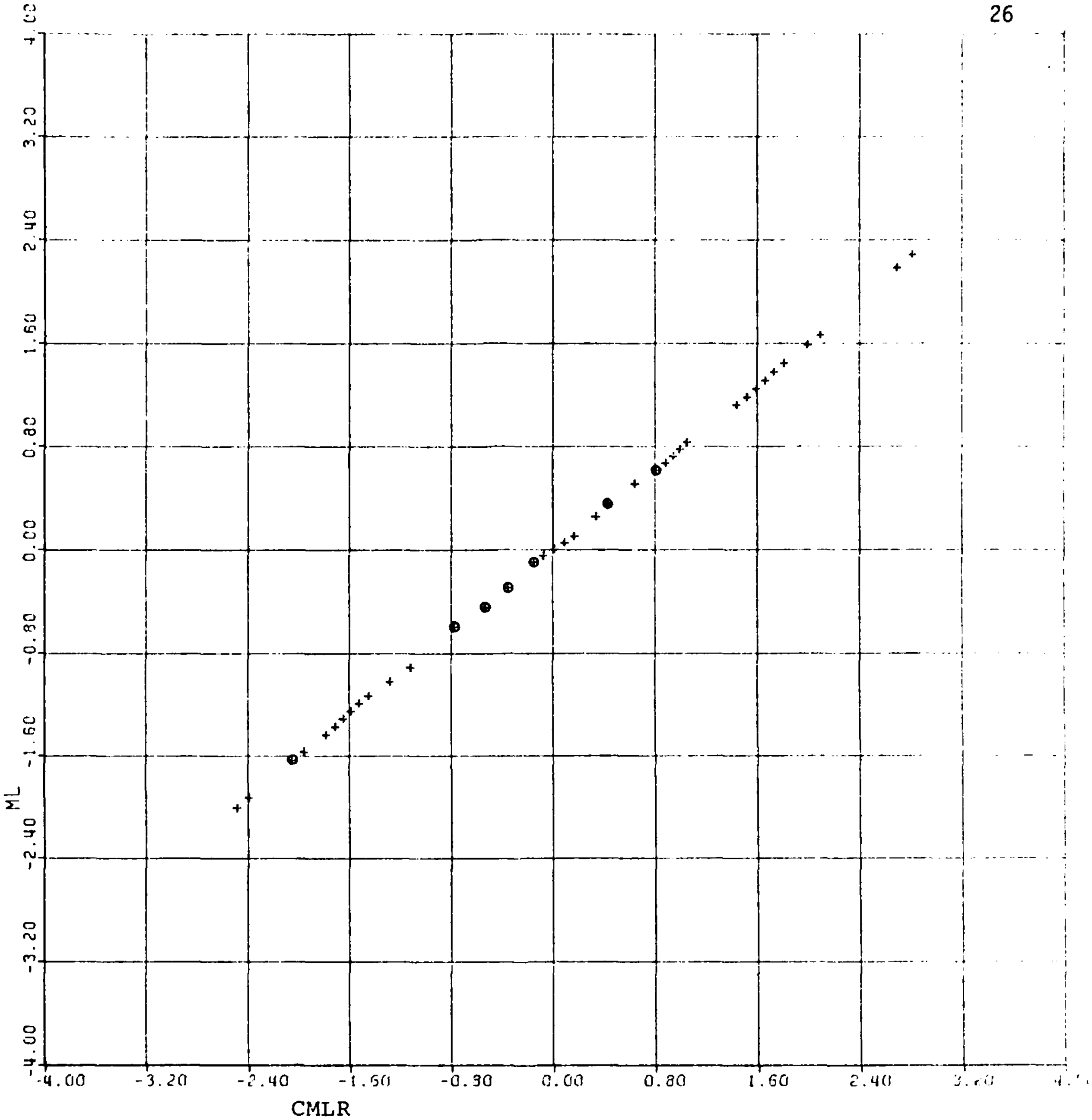

FIGURE 7. ML vs CMLR Estimates of Difficulty Parameters

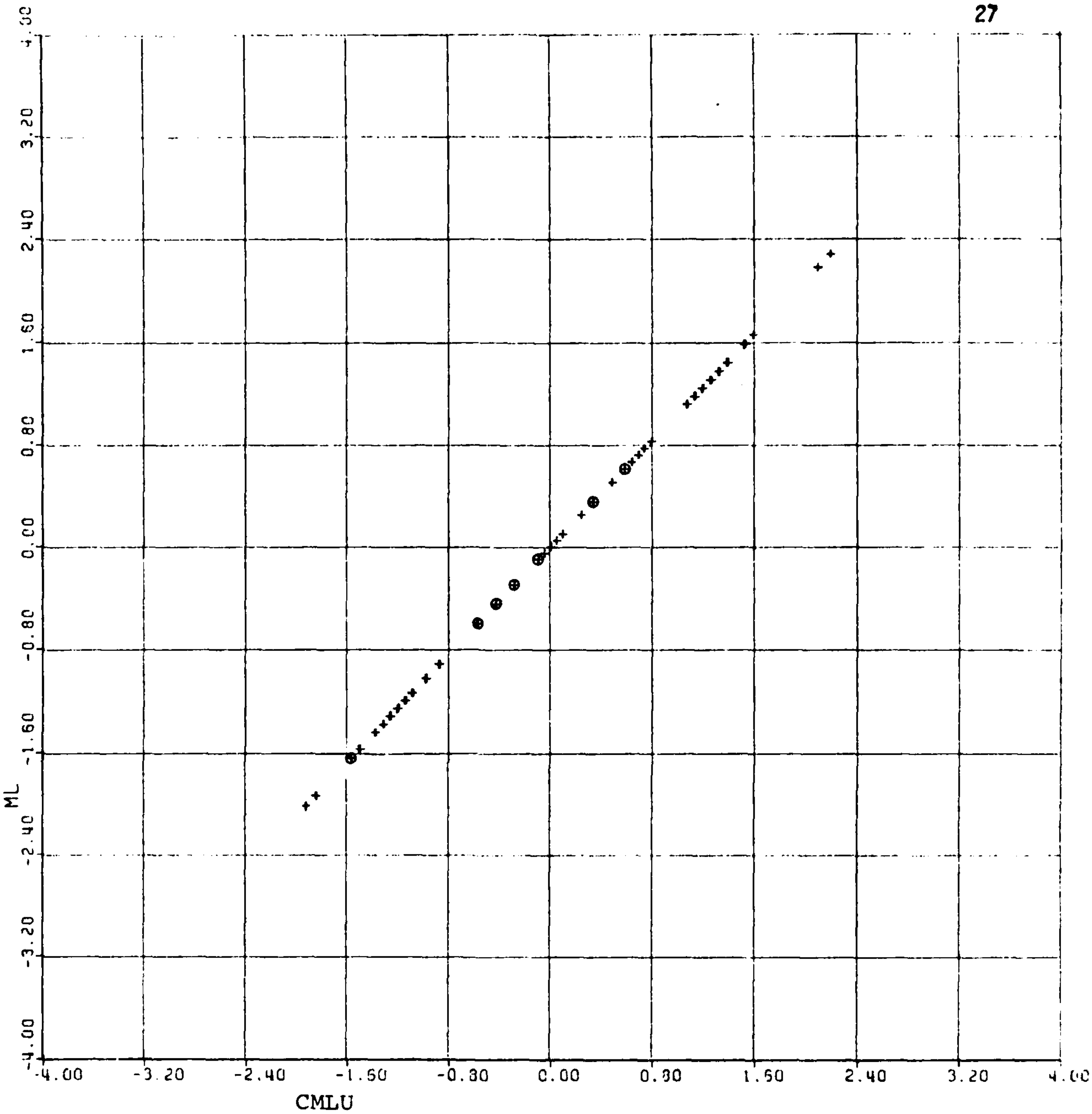

FIGURE 8. ML vs CMLU Estimates of Difficulty Parameters

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